Optimum Number of Teeth for Span Measurement

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Abstract

An expression is derived, giving the optimum number of teeth over which the span measurement should be made, for profile-shifted spur and helical gears.

Introduction

The span measurement is widely accepted as an accurate method for measuring gear tooth thickness. Once the span has been measured, for either spur or helical gears, the tooth thickness can be calculated without difficulty.[1] However, there is no simple expression, except in the case of spur gears with zero profile shift, giving the optimum number of teeth N′ over which the span should be measured.

When the span measurement is made, the contact between the tooth faces and the caliper jaws should be near the middle of the tooth profile. For gears with no profile shift, this means that the contact should be near the standard pitch circle. And for gears with profile shift e, the radius of the tip circle is generally extended by approximately the same amount e, so the contact should take place at distance e above the standard pitch circle. It is not possible to choose N′ so that the contact is always close to the required radius. For certain gears, one value of N′ may give contact points near the tips of the teeth, while, if N′ is reduced by 1, the contact points may be down near the fillets. However, it is obviously essential that the contact should always take place below the tooth tips and above the fillets, preferably with an adequate margin.

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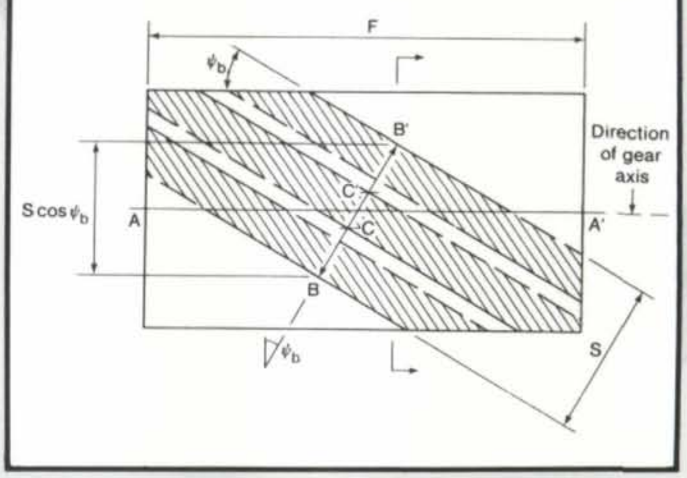
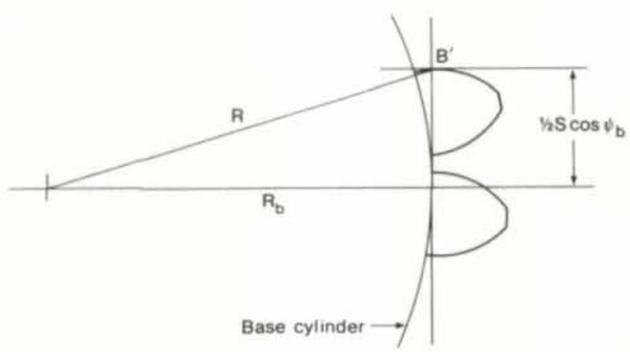
Fig. 1—A base tangent section



Relation Between the Span Measurement and the Tooth Thickness

Fig. 1 shows a base tangent section through the gear, touching the base cylinder along line AA′. The sections through each tooth are indicated by the shaded areas in the diagram. The upper edge of each tooth coincides with a generator above line AA′ and is, therefore, a straight line; while below line AA′, the tooth edge in the section is slightly curved. Similarly, the lower edge of each tooth is curved above line AA′, and it is straight below the line. Since the curved edges are very nearly straight, they are shown in the diagram as straight dotted lines. The span measurement is made between two points such as B and B′, on opposite sides of line AA′, and because B and B′ each lie on a straight tooth edge, the exact shape of the curved edges is of no significance. With the curved edges drawn as straight lines, the tooth edges all

Fig. 2—Transverse section through a contact point



appear as parallel straight lines, making an angle with the gear axis equal to the base helix angle ψ_b, and the base tangent section is then identical with the developed base cylinder.

The length BB′ is made up of one tooth thickness measurement CC′, *which crosses line AA′, and a number of pitch* measurements such as BC. Since the base tangent section is the same as the developed base cylinder, at least as far as the solid lines are concerned, the length CC′ is equal to t_{nb}, the normal tooth thickness at the base cylinder, and the length BC is equal to the normal base pitch p_{nb}. Hence, if the span S is measured over N′ teeth, there are (N′−1) lengths such as BC, and the span length is given by the following expression:

$$S = (N' - 1)p_{nb} + t_{nb} \qquad (1)$$

This equation gives the value of t_{nb} immediately. The tooth thickness at the standard pitch cylinder can be found by converting first to the transverse tooth thickness at the base cylinder, and then to that at the standard pitch cylinder, in the usual manner.

$$t_{nb} = S - (N' - 1)p_{nb} \qquad (2)$$

$$t_{tb} = \frac{t_{nb}}{\cos \Psi_b} \qquad (3)$$

$$t_t = R_s \left(\frac{t_{tb}}{R_b} - 2 \text{ inv } \emptyset_s\right) \qquad (4)$$

$$t_n = t_t \cos \Psi_s \qquad (5)$$

In these equations, all quantitites with the subscript b are defined on the base cylinder, while the others are defined on the standard pitch cylinder, whose radius R_s is defined as follows:

$$R_s = \frac{N\, m_n}{2 \cos \Psi_s} \qquad (6)$$

where m_n is the normal module. The equations in this article are all given in terms of the normal and transverse modules m_n and m_t, but for those who prefer to use the corresponding diametral pitches, the final expression for the number of teeth to be spanned is given in both forms.

Optimum Span Length

The component of the span measurement perpendicular to the gear axis is (S cos Ψ_b), as we can see in Fig. 1. The transverse section through one of the measurement contact points is shown in Fig. 2, and the radius R of this point is found by Pythagoras' rule:

$$R^2 = R_b^2 + (\tfrac{1}{2}S \cos \Psi_b)^2 \qquad (7)$$

The profile shift e is related to the tooth thickness by the following two equations:

$$t_t = \tfrac{1}{2}\pi\, m_t + 2e \tan \emptyset_s \qquad (8)$$

$$t_n = \tfrac{1}{2}\pi\, m_n + 2e \tan \emptyset_c \qquad (9)$$

where $\emptyset_s$ and $\emptyset_c$ are the transverse and normal pressure angles of the gear at its standard pitch cylinder. For a gear with zero profile shift, the radius of the tip circle is generally *approximately equal to* $(R_s + m_n)$, *while in a gear with pro-*file shift e, this radius is usually increased by approximately e. Hence, if we want the contact to take place at a distance m_n below the tip circle, the radius R should be as close as possible to the radius of the standard pitch cylinder, plus the profile shift e.

$$R \cong R_s + e \qquad (10)$$

We substitute this expression for R into Equation (7), and replace the base cylinder radius R_b by ($R_s \cos \emptyset_s$), in order to obtain an expression for the optimum span measurement.

$$(\tfrac{1}{2}S \cos \Psi_b)^2 \cong (R_s + e)^2 - (R_s \cos \phi_s)^2 \qquad (11)$$

This equation is simplified by expanding the righthand side as a power series in (e/R_s), and retaining only the first two terms.

$$\tfrac{1}{2}S \cos \Psi_b = R_s \sin \phi_s + \frac{e}{\sin \phi_s} \qquad (12)$$

Number of Teeth to be Spanned

An expression for S was given in Equation (1). Before we make use of the expression, we put it in a more convenient form. The normal base pitch is expressed in terms of, first the transverse base pitch, then the transverse pitch at the standard pitch cylinder, and finally the transverse module. In a similar way, the tooth thickness t_{nb} is expressed in terms of t_{tb}, and then by means of Equations (4 and 8), in terms of the transverse module and the profile shift. We then obtain the following expression for S:

$$S = \cos \Psi_b \cos \phi_s \, [(N' - 1)\, \pi m_t + \tfrac{1}{2}\pi m_t + 2e \tan \phi_s + N m_t \,\mathrm{inv}\, \phi_s] \qquad (13)$$

We substitute this expression into Equation (12), and solve for N', the optimum number of teeth over which the span should be measured.

$$N' = \tfrac{1}{2} + \frac{N\phi_s}{\pi} + \frac{N}{\pi} \tan \phi_s \tan^2 \Psi_b + \frac{2e}{\pi\, m_n \tan \phi_c} \qquad (14)$$

The term $(N\phi_s/\pi)$ is derived from the function inv ϕ_s in the expression for S, and the angle ϕ_s must, therefore, be expressed in radians. Since it is common practice to use degrees in equations of this sort, the term can be replaced by $(N\phi_s^\circ/180)$.

In general, the value of N' given by Equation[14] is not an integer. It is obvious that the span measurement can only be made over an integer number of teeth, so in order to keep the measurement contact radius as close as possible to $(R_s + e)$, we choose N' as the integer closest to the value given by Equation.[14] Since the real value of N' may, therefore, differ by as much as 0.5 from the ideal value, it is important to determine whether the contact radius is still satisfactory.

For any particular gear, we can calculate the contact radius R by first finding the integer value N', then substituting this into Equation (13) to obtain the span length S, and finally using Equation[7] to find the corresponding value of R. We can, therefore, verify the validity of the expression for N' by calculating the contact radius R for a large number of gears. To be satisfactory, the radius R must always be less than the tip circle radius R_T, and greater than R_f, the radius of the true involute form circle which passes through the tops of the fillets.

The tip circle radius is chosen by the designer, but is generally close to the following value:

$$R_T = R_s + e + m_n \qquad (15)$$

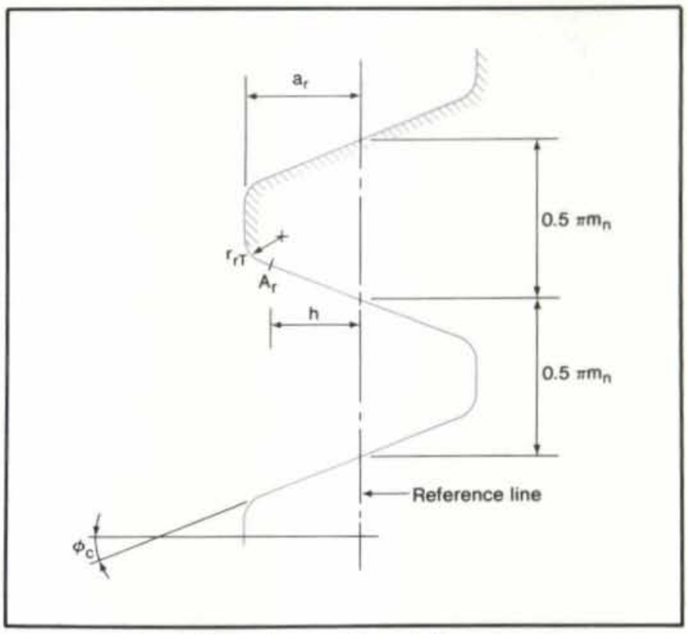


Fig. 3 – Normal section through the cutter

The radius of the true involute form circle depends on the type of cutter, and the dimensions of its teeth. For the present purpose, we will assume that the gear is cut by a rack cutter, with the normal tooth section shown in Fig. 3. The normal pitch is πm_n, and the normal pressure angle is ϕ_c. The reference line is the line along which the tooth thickness is 0.5 πm_n, the addendum measured from this line is a_r, and the tooth tip radius is r_{rT}. If A_r is the end point of the straight section of the tooth profile, the distance h of point A_r from the reference line is given by the following expression:

$$h = a_r - r_{rT}\,(1 - \sin \phi_c) \qquad (16)$$

Fig. 4 shows the transverse section through the gear and

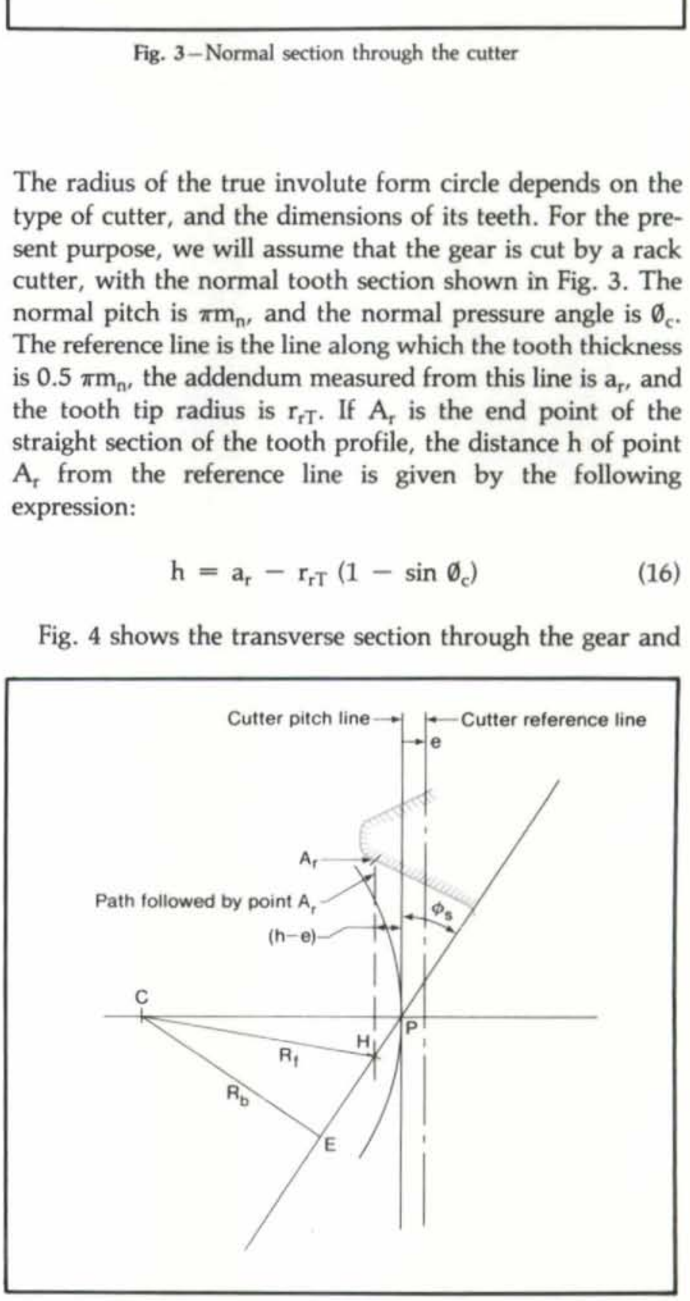


Fig. 4 – Transverse section through the gear and cutter

cutter during the cutting process. If the gear has a profile shift e, the cutter is offset by the same amount e, and the path followed by point A_r of the cutter is, therefore, a distance (h-e) from the pitch point. On the gear tooth, the end point of the fillet is cut by point A_r of the cutter, when A_r reaches the line of action. The radius R_f of the true involute form circle on the gear can then be found from the dimensions shown in Fig. 4:

$$R_f^2 = CE^2 + EH^2 = R_b^2 + \left[R_b \tan \emptyset_s - \frac{h-e}{\sin \emptyset_s}\right]^2 \quad (17)$$

For the calculations described later in this article, it is assumed that the length h is equal to the normal module m_n.

If we simply wanted to be sure that we have a suitable value of N' for one particular gear, we could calculate the value of the contact radius R, and check R is less R_T and greater than R_f. In order to verify the general expression for N', we must carry out the same check for a very large number of gears, and it is essential to include all the worst possible cases. We define a measure ΔR of the error in N', as the difference between the actual and the ideal contact radii:

$$\Delta R = R - (R_s + e) \quad (18)$$

It is clear that the contact is only close to the tip circle or the true involute form circle in cases where the magnitude of ΔR is large.

We will consider gears with profile shift values between $-0.5\ m_n$ and $1.0\ m_n$. If we choose particular values for $\emptyset_c$, Ψ_s and N, and calculate the error ΔR for various values of e, we obtain a function such as the one shown in Fig. 5. The discontinuities occur each time there is a change in the value of N'. Since N' is the integer closest in value to the expression in Equation (14), the discontinuities in ΔR occur whenever this expression is exactly midway between two integers. The magnitude of ΔR reaches its largest values just before and just after each discontinuity, so these are the values of e at which the error must be calculated. In addition, it should also be calculated at $e = -0.5m_n$ and $e = 1.0\ m_n$.

The calculations just described have been carried out for various values of $\emptyset_c$, Ψ_s and N, with the following results.

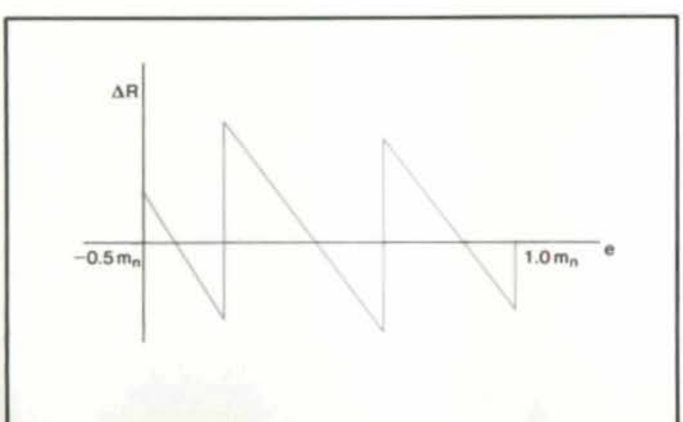


Fig. 5—Contact position error as a function of profile shift

The contact radius is satisfactory in all gears with small values of e, but for gears with large amounts of profile shift the value of N' given by Equation (14) is sometimes too large, and the contact points are then too close to the tooth tips. This occurs particularly in gears with a small number of teeth. In a few cases, the value of R is actually larger than R_T.

The reason for this large error is that the power series expansion used to derive Equation (12) is only accurate when (e/R_s) is small. The addition of a third term in the expansion makes the expression for N' very much more complicated, and does not significantly improve the accuracy. We, therefore, alter the expression in a manner that reduces the value of N' whenever e is large. We multiply the coefficient of e by the factor $[0.75 - (2/N)]$, which has the effect of halving the coefficient when N is 8, and reducing it by about 25% when N is large.

The number N' is then the integer closest to the following expression:

$$N' = \tfrac{1}{2} + \frac{N\emptyset_s^\circ}{180} + \frac{N}{\pi} \tan \emptyset_s \tan^2 \Psi_b + \frac{2e[0.75-(2/N)]}{\pi m_n \tan \emptyset_c} \quad (19)$$

Since a span measurement over one tooth is impossible, the minimum value to be used for N' is 2, even in cases where the value of the expression in Equation (19) is less than 1.5. The equation can also be expressed in terms of the normal diametral pitch P_{nd}, if the ratio (e/m_n) is replaced by the quantity (eP_{nd}).

There is no theoretical basis for the modification made to the coefficient of e. However, the new expression for N' has been tested in the manner described earlier, and has been found to work very well. The calculations were carried out for normal pressure angles of 14.5°, 15°, 17.5°, 20°, 22.5° and 25°, helix angles of all integers from 0° to 45°, and all tooth numbers from 8 to 160.

In the cases considered, the minimum distance between the contact point and the tooth tip was 0.383 m_n, while the minimum distance between the contact point and the top of the fillet was 0.272 m_n. With the ranges of $\emptyset_c$, N and e values that were studied, it is clear that some of the gears would be undercut. In order to avoid making calculations for gears that would never exist in practice, the calculation was discontinued in cases where the ideal contact radius (R_s + e) was less then R_b. For the remaining undercut gears, the radius of the contact point was compared with the undercut circle radius, which can be found by trial and error.[2] The minimum distance between the two circles was found to be 0.133 m_n.

We have shown that, for gears with standard length teeth, there is always adequate clearance between the measurement contact point and the tooth tip. Equation (19) can, therefore, also be used to calculate the optimum value of N' for stub-toothed gears. In addition, the clearance is generally sufficient to allow the span measurement of gears with tip relief. The contact takes place either on the involute part of the tooth face, or on a part where the tooth thickness is not significantly altered by the tip relief. The only exceptions occur in gears with a very small number of teeth, for which the span measurement is made over only two teeth. In some of these gears, the contact point may lie well within the tip-relieved part of the tooth face. It is, therefore, advisable for gears with N' equal to 2, to check the value of R at which the measurement contact is made. If the tooth thickness at this radius is seriously reduced by the tip relief, then the span measurement should not be used.

Examples

The first example deals with the inspection measurement of a typical helical gear, and shows how Equations (19, 13 and 7) are used to calculate the number of teeth to be spanned, the corresponding span length, and the radius of the measurement contact point.

Given: $m_n = 10$ mm, $N = 35$, $\emptyset_c = 20°$, $\Psi_s = 30°$,

$e = 3.000$ mm

Then: $$R_s = \frac{Nm_n}{2\cos\Psi_s} = 202.073$$

$$m_t = \frac{m_n}{\cos\Psi_s} = 11.547$$

$$\tan\emptyset_s = \frac{\tan\emptyset_c}{\cos\Psi_s} \qquad \emptyset_s = 22.796°$$

$$R_b = R_s\cos\emptyset_s = 186.289$$

$$\tan\Psi_b = \frac{R_b\tan\Psi_s}{R_s} \qquad \Psi_b = 28.024°$$

N' = Integer closest to (6.6225) = 7

S = 201.312 mm

R = 206.394 mm

R − (R_s + e) = 1.322 mm

The second example deals with a spur gear, and was chosen to illustrate that we obtain a suitable measurement radius when N' is calculated from Equation (19), but that the earlier expression given by Equation (14) sometimes gives an impractical measurement radius.

Given: m_n = 10 mm, N =, $\emptyset_c$ = 14.5°,
Ψ_s = 0, e = 9.000 mm;

Then: $$R_s = \frac{Nm_n}{2} = 60.000$$

$m_t = m_n$, $\emptyset_s = \emptyset_c$, $\Psi_b = 0$
$R_b = R_s\cos\emptyset_s = 58.089$

Using Eq. (19), N'= Integer closest to (2.7590) = 3
S = 81.189 mm
R = 70.868 mm
R − (R_s + e) = 1.868 mm

Using Eq. (14), N' = Integer closest to (3.6821) = 4
S = 111.604 mm
R = 80.549 mm
R − (R_s + e) = 11.549 mm
$R_T = R_s + e + m_n = 79.000$ mm
i.e. $R > R_T$

References

1. M. F. SPOTTS, "Special Calculations for Spur and Helical Gears", Chapter 7 in the "Gear Handbook", edited by D. W. Dudley, McGraw-Hill, 1962.
2. J. R. COLBOURNE, "The Design of Undercut Gears", ASME paper 80-C2/DET-67, presented at the Century 2 International Power Transmissions and Gearing Conference, San Francisco, 1980.

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