New Approach to

By Veniamin I. Goldfarb and Alexander A. Tkachev

Computerized Design of Spur and Helical Gears

Management Summary

In gear design, choosing shift coefficients x_1 and x_2 for a pinion and gearwheel may be crucial to providing the necessary quality for spur and helical gears. This choice can be made by means of a so-called dynamic blocking contour (DBC), which is described in this paper. The contour contains important information about a number of quality parameters for a gear design and can be drawn with help from a "Contour" CAD system when choosing a gear's initial parameters.

When applying a DBC, the designer can predict the gear's quality at the earliest stage of its design, i.e. when choosing the values x_1 and x_2. This ability allows a designer to increase considerably the productivity and quality of the gear design process, eliminating complex iterative procedures for finding optimal solutions.

Introduction

One effective way to enhance gear design is to allow users to forecast the quality of a gear at the initial design stage—that is, when choosing the basic, initial geometric parameters. Such opportunity decreases considerably the range of variable parameters and raises the design quality and efficiency.

This idea is implemented in a new approach to the design of spur and helical gears. The approach is based on the application of a special type of geometrical objects named blocking contours (Refs. 1–4). These BCs are used to choose rational values for addendum modification coefficients (profile shift coefficients) of a pinion and gearwheel, x_1 and x_2, respectively. The influence of shift coefficients on the calculation of a gear's tooth geometry and kinematic and strength parameters is widely known. Given this influence, many properties of a gear can be estimated at the stage of x_1 and x_2 selection by means of blocking contours—that is, at the initial design stage. Thus, the process of choosing optimal gear parameters is simplified considerably.

Based on the BC concept, computer-aided design of spur and helical gears has been developed to achieve the stated design principles. In order to better illustrate the design, CAD shows the meshing processes of the gear pair and shows meshing element generation by a rack-type cutting tool. Release of the developed system allows users to master the methodology and possibilities of gear design using the BC concept to evaluate the selection of shift coefficients to obtain specified gear properties.

Dr. Veniamin I. Goldfarb

is a professor in the production mechanical engineering department of Izhevsk State Technical University, located in Izhevsk, Russia. He's also the department's head and is director of the university's Institute of Mechanics. Goldfarb specializes in the theory and practice of gear design, investigation and production. In addition, he's chair of the gearing technical committee of the International Federation for the Theory of Machines and Mechanisms.

Dr. Alexander A. Tkachev

is an assistant professor in the university's production mechanical engineering department, but he's mainly responsible for programming gear design software, especially for designing spur and helical gears. Tkachev is also executive engineer of Izhevsk Mechanical Plant, a company that produces power tools and other products. As executive engineer, he is involved in developing CAD systems for gears.

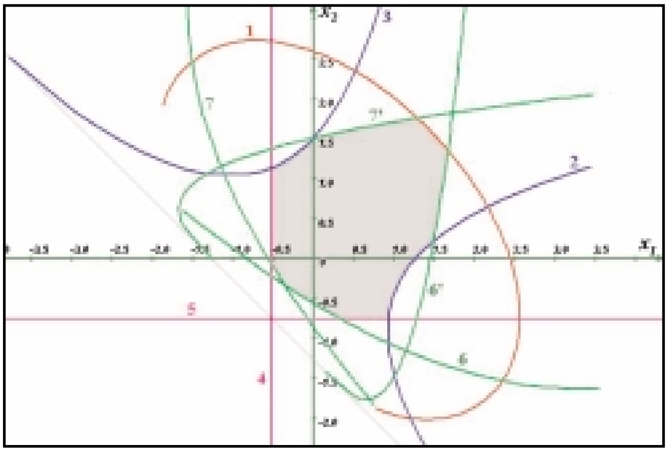


Figure 1—View of a blocking contour.

Essence of the Blocking Contours Method

A set of lines is plotted on the coordinate plane x_1, x_2, the lines corresponding to certain values of basic factors which limit the valid process of meshing. These values can include maximum allowable ones. The factors are:

1.) the absence of undercut when cutting pinion and gearwheel teeth,

2.) the absence of interference in operating meshing,

3.) a transverse contact ratio ε_α—including its allowable value, for example $\varepsilon_\alpha = 1$, and

4.) tooth thickness at the addendum circle s_a and its minimum allowable value, for example $s_a = 0$, corresponding to sharp or pointed gear teeth.

When intersecting with each other, these lines form a closed area on the coordinate plane. The point (x_1, x_2) for the pinion (x_1) and wheel (x_2) must be within this area, otherwise one or more of the gear operation criteria stated above will not be true. Figure 1 shows limiting lines in terms of undercut (straight lines 4 for a pinion and 5 for a wheel), interference (two lines for a pinion, 6 and 6', and two for a wheel, 7 and 7'), transverse contact ratio $\varepsilon_\alpha = 1$ (line 1) and the sharp teeth of the pinion ($s_{a1} = 0$, line 2) and wheel ($s_{a2} = 0$, line 3). The area defined by these lines (the figure's shaded area) is the domain of coefficients x_1 and x_2 which meet the above criteria. This shaded area is bounded by parts of the abovementioned limiting lines 1–7'. These parts make up a closed line. This line is called a blocking contour (BC).

To determine the limiting lines, the following mathematical equations should be used.

1.) Lines for the undercut are straight lines parallel to the coordinate axes:

$$x_i = h_l^* - h_a^* - \frac{z_i \sin^2\alpha_t}{2\cos\beta} \qquad (1)$$

where $i = 1$ for a pinion, $i = 2$ for a gearwheel;
h_l^* is a boundary height factor equal to the ratio of the height of the rectilinear section of an initial contour ("producing, or basic rack") to the module m; h_a^* is an addendum factor, one of the main basic rack parameters (in most cases $h_l^* = 2h_a^*$, the standard value $h_a^* = 1$);
z_i is the tooth number (z_1 is for a pinion, z_2 is for a gearwheel);
β is the helix angle ($\beta = 0°$ for spur gears);

$$\alpha_t = \arctan\left(\frac{\tan\alpha}{\cos\beta}\right)$$

is the pitch profile angle in lateral cross-section; here α is the pressure angle, another important basic rack parameter.

2.) Lines of interference.

For a pinion:

$$a_w \sin\alpha_{tw} - 0.5d_{b2}\tan\alpha_{a2} = 0.5d_1\sin\alpha_t - \frac{h_l^* - h_a^* - x_1}{\sin\alpha_t} m \qquad (2)$$

where
a_w is the gear center distance;
α_{tw} is the gear pressure angle

$$\text{inv}\alpha_{tw} = 2\frac{x_1 + x_2}{z_1 + z_2}\tan\alpha + \text{inv}\alpha_t \qquad (3)$$

where "inv" is the designation of an involute function of an angle ($\text{inv}\alpha = \tan\alpha - \alpha$), sometimes also called an involute angle; a_w and α_{tw} are interconnected with a known relation

$$a_w = a\frac{\cos\alpha_t}{\cos\alpha_{tw}}$$

where $$a = \frac{(z_1 + z_2)m}{2\cos\beta}$$

is the pitch center distance;
d_{b2} is the gearwheel base diameter;
α_{a2} is the tooth profile angle of a gearwheel at the point of tooth addendum circumference

$$\cos\alpha_{a2} = \frac{d_{b2}}{d_{a2}}$$

where d_{a2} is the gearwheel tip diameter (diameter of the addendum circle); and
formulae for d_{bi} and d_{ai} are well known.

With these formulae, one can obtain the expressions for $\cos\alpha_{a1}$ and $\cos\alpha_{a2}$, which include shift coefficients x_1 and x_2:

$$\cos\alpha_{a1,2} = \frac{z_{1,2}\cos\alpha_t}{2(h_a^* - x_{2,1})\cos\beta + (z_1 + z_2)\dfrac{\cos\alpha_t}{\cos\alpha_{tw}} - z_{2,1}} \quad (4)$$

d_1 is a pinion reference diameter ($d_1 = mz_1$).

In the equation for a gearwheel, indices 1 and 2 interchange their places.

Using the known relations for a_w, d_{b2} and d_1 in equation 2, we can obtain the following correlation for the pinion:

$$(z_1 + z_2)\cos\alpha_t\tan\alpha_{tw} - z_2\cos\alpha_t\tan\alpha_{a2} = z_1\sin\alpha_t - 2\,\frac{\cos\beta}{\sin\alpha_t}\,(h_l^* - h_a^* - x_1) \quad (5)$$

where α_{tw} and α_{a2} depend upon x_1 and x_2 according to equations 3 and 4.

3.) Line of transverse contact ratio ε_α

$$\frac{1}{2\pi}[z_1\tan\alpha_{a1} + z_2\tan\alpha_{a2} - (z_1 + z_2)\tan\alpha_{tw}] = \varepsilon_\alpha \quad (6)$$

where ε_α is a given value of transverse contact ratio, e.g. $\varepsilon_\alpha = 1$.

4.) Lines of tooth thickness at the addendum circle s_{ai} (i = 1 or 2, as before):

$$s_{ai} = d_{ai}\left(\frac{\pi}{2z_i} + \frac{2x_i\tan\alpha}{z_i} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{ai}\right) = k_{si}m \quad (7)$$

The value k_{si} determines the tooth thickness at the addendum circle proportional to the module. For the minimum allowable value k_{si} = 0, the expression defines the line which is the boundary limited by sharp or pointed teeth.

Using the known expression for d_{a1}, we can obtain the following formula for the pinion:

$$\left[2(h_a^* - x_2) + \frac{z_1 + z_2}{\cos\beta}\,\frac{\cos\alpha_t}{\cos\alpha_{tw}} - \frac{z_2}{\cos\beta}\right] \bullet \left[\frac{\pi}{2z_1} + \frac{2x_1\tan\alpha}{z_1} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{a1}\right] = k_{s1} \quad (8)$$

which, for a given k_{s1}, is the equation of the line s_{a1} in the plane x_1, x_2.

The equation of the line s_{a2} for the gearwheel is almost the same, indices 1 and 2 interchanging their places.

Equations 1–8 have been used for calculation and construction of BC limiting lines in the coordinate plane x_1, x_2. For more fast and reliable calculation of the lines, special algorithms were developed and used in a "Contour" CAD system (see the "Example" section below) and proved to be correct and effective. This system is a gear CAD system created by the authors based on the BC concept.

The lines ε_α, s_{a1} and s_{a2}, which are among the mentioned BC curves, depend on initial parameters such as tooth number z_1 and z_2 for a pinion and gearwheel respectively, helix angle β and basic rack parameters—in particular, pressure angle α and addendum factor h_a^*. They also depend on values of parameters for which these lines are constructed. When designing a gear, the values of these latter parameters are taken either from standards or from practice. For example, the minimum allowable value of ε_α is chosen as 1.0, 1.1 or 1.2 in accordance with different standards and the type of gear being designed (spur or helical); the minimum allowable value of $s_{a1,2}$ = 0.25m or 0.4m. In the practice of design and research, however, it is often necessary to operate with non-standard values for these parameters. Moreover, in order to increase the flexibility of the design process, one may need to simultaneously construct several lines of one type with different values for the determining parameter. This was the reason for the development of the dynamic blocking contour (DBC) concept.

Let's consider the case when an engineer designs a helical gear and wants the transverse contact ratio to be within the range 1.0 ≤ ε_α ≤ 1.4. Then the BC configuration containing two transverse contact ratio lines ε_α = 1.0 and ε_α = 1.4 (represented by lines 1 and 1' in Fig. 2) is required. If the engineer takes shift coefficients [point (x_1, x_2)] within the area between these curves (the figure's shaded area), the value of ε_α is guaranteed to be within the given range.

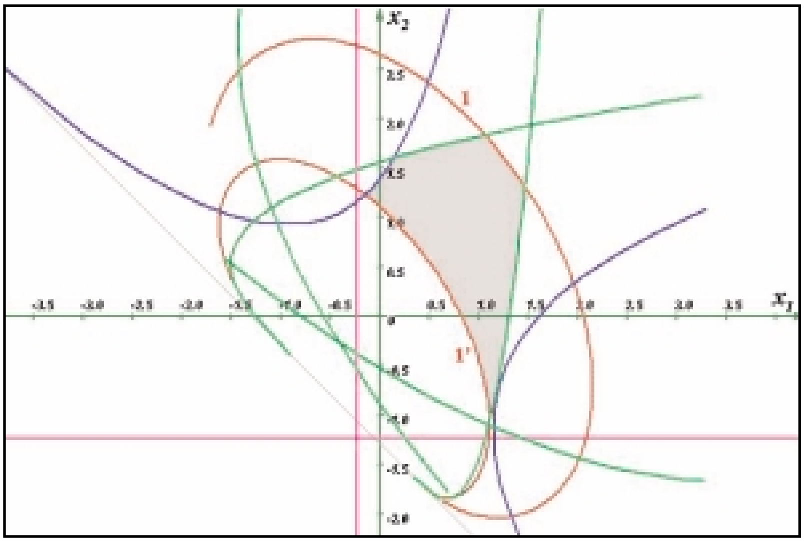


Figure 2—Blocking contour with $\varepsilon_\alpha 1$ = 1.0 (line 1) and $\varepsilon_\alpha 1$ = 1.4 (line 1').

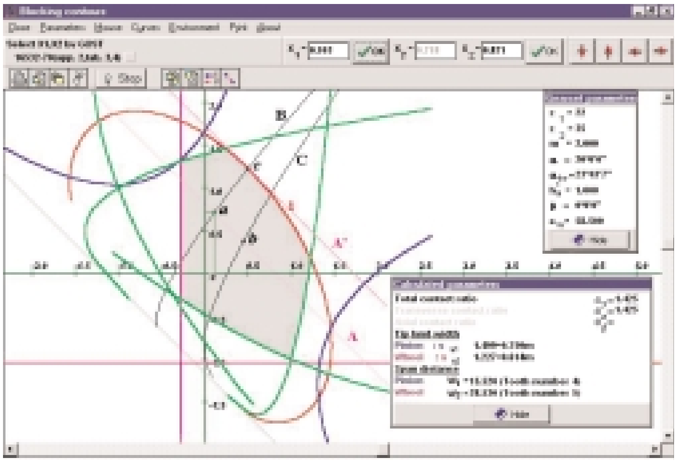


Figure 3—Description of features of BC lines.

Some Features of Blocking Contour Lines

A blocking contour (BC) has a property which becomes very important for implementing the abovementioned concept of gear design. The property is: In order to calculate and construct a BC, it is necessary to have certain minimum information, which is usually known at the first design stage:

1.) tooth number of pinion z_1 and gearwheel z_2;
2.) initial contour (producing rack) parameters—in particular, profile angle α, tooth addendum coefficient h_a* and radial clearance coefficient c*; and
3.) helix angle β (for helical gears).

In the simplest case, when designing a spur gear (β = 0°) with "standard" initial contour (α = 20°, h_a* = 1, c* = 0.25), it is necessary to know only values z_1 and z_2. Note here that the geometric configuration of a BC does not depend on gearing module m.

In order to apply a BC more effectively, "additional" lines within the coordinate plane x_1 and x_2 are used along with the abovementioned lines generating the blocking contour. These lines, like the lines forming the BC, are the geometrical representation of several properties of a gear under design. Let's consider some of these lines (Fig. 3).

I. Straight line with the equation $x_1 + x_2 = x_\Sigma$ (line A in Fig. 3)

Here x_Σ is the so-called coefficient of shifts sum. This line is important in the practical operation of a blocking contour. When the value of the shifts sum coefficient is fixed (x_Σ = constant), we obtain constant values of pressure angle α_{tw} and center distance a_w according to formulas

$$\text{inv}\alpha_{tw} = 2\frac{x_\Sigma}{z_1 + z_2}\tan\alpha + \text{inv}\alpha_t \quad \text{and}$$

$$a_w = a\frac{\cos\alpha_t}{\cos\alpha_{tw}}$$

which appeared earlier.

From a geometrical point of view, it means that:

1.) the position of the line $x_1 + x_2 = x_\Sigma$ in the coordinate plane x_1, x_2 is defined by the value of one of three unambiguously interconnected parameters x_Σ, α_{tw} or a_w;
2.) pressure angle α_{tw} and center distance a_w remain constant when displacing the point (x_1, x_2) along the stationary straight line $x_1 + x_2 = x_\Sigma$ (that is along the line with the fixed x_Σ = constant). In practical terms, this is important because in actual gear design, center distance a_w is often predetermined and can't be changed.

II. Line with the equation $\alpha_{a1} = \alpha_{a2}$ (line B in Fig. 3)

Here α_{a1} and α_{a2} are tooth profile angles of a pinion and a gearwheel, respectively, at the points of tooth addendum circumference. As research showed, this line has two following interconnected properties:

1.) for a given center distance a_w = constant, the maximum transverse contact ratio ε_α is achieved at the point (x_1, x_2) of intersection of the given line with line A ($x_1 + x_2 = x_\Sigma$) compared with other points (x_1, x_2) of line A. Since the coefficient ε_α is known to be one of the main parameters defining smoothness of gear operation, the line under consideration may be called "the line of maximum smoothness";
2.) for a given transverse contact ratio ε_α, the maximum possible value of the shifts sum coefficient x_Σ (and therefore the maximum values of a_w and α_{tw}) is achieved along line B, at the point of its intersection with the line ε_α (line 1 in Fig. 3). As the value x_Σ increases, the values of curvature radii for the pinion and gearwheel tooth involute profiles also increase, leading to a decrease of contact stresses and, therefore, an increase of gear contact strength. This opportunity, however, can be practically implemented only

when the center distance a_w is variable within a wide range, which happens very seldom. That is why this line is alternately called "the line of maximum contact strength" or, at least, "the line of *increased* contact strength." The line may also be called "the line of minimum contact stresses."

III. Line of equal specific sliding (line C in Fig. 3)

This line has the equation (Ref. 5)

$$u\left(\frac{\tan\alpha_{tw}}{\tan\alpha_{a2}}-1\right)=\frac{\tan\alpha_{tw}}{\tan\alpha_{a1}}-1 \qquad (3)$$

where $u = \frac{z_2}{z_1}$

is the gear ratio. If the shift coefficients x_1 and x_2 are chosen so the point (x_1, x_2) could belong to this line (or, at least, to its vicinity), the gear operating conditions will be favorable for reducing the risk of scuffing and abrasive wear.

Moreover, it is emphasized in Reference 1 that such choice of x_1, x_2 provides more beneficial values of tooth shape coefficient Y_F from a bending strength point of view.

Therefore, it is possible to choose the values of x_1, x_2 at the initial stage of gear design by means of a BC and to choose lines A, B and C described above to solve the following tasks:

- the given value of center distance a_w (choice of the position of line A in the plane x_1, x_2 is provided);
- maximum smoothness of gear operation (point of intersection of lines A and B) is achieved for a given a_w, or minimum risk of scuffing and abrasive wear is achieved by equalizing of specific sliding (point of intersection of lines A and C);
- minimum contact stress is achieved for the chosen overlap ratio ε_α at the point of intersection of line B with line 1 corresponding to the given value ε_α. The user immediately obtains the displayed values of x_Σ, a_w and ε_α corresponding to the chosen point (x_1, x_2) where contact stress is noted to be minimum.

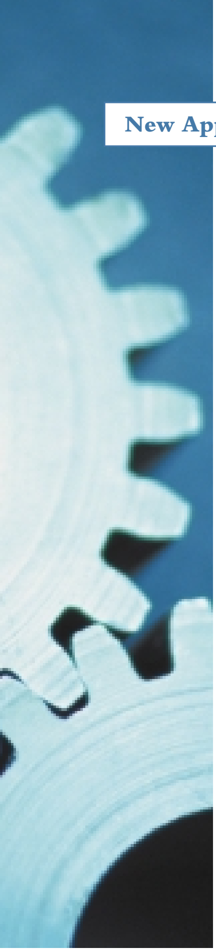
Thus, the concept of gear property evaluation and selection of the values x_1, x_2 at the initial design stage is provided by means of blocking contours with the lines A, B and C described above. Later in the gear design stage, the corresponding properties can be proved by calculations. Here the designer avoids performing time-consuming iterative procedures when choosing optimal shift coefficients of a pinion and gearwheel.

Example

Let's consider the blocking contours of an external spur gear (Fig. 3) presented to the user of a "Contour" CAD system based on the described concept. A spur gear with the following initial parameters is considered: z_1 = 22, z_2 = 35, m = 2 mm; α = 20° and h_a* = 1. (This data is presented in a small panel "Gearset parameters" near the figure's upper right corner.) Center distance a_w is assumed to be predetermined a_w = 58.5 mm. The area limited by the blocking contours is shaded in the figure. Line A corresponds to the initial center distance a_w = 58.5 mm. The shift coefficients at point *a*, where lines A and B intersect, are equal to x_1 = 0.103, x_2 = 0.718 and x_Σ = 0.821. The maximum allowable transverse contact ratio ε_α = 1.425 for the given a_w is achieved at this point. (This ratio is shown in the panel "Calculated parameters" in the figure's lower right corner.)

If the point (x_1, x_2) is displaced along line A to the right and downwards to position b—that is, to the intersection with line C—then x_1 = 0.445 and x_2 = 0.376. For these coefficients, the advantage of the line of equal specific sliding will be obtained (see above). It is clear that choosing the position of the point (x_1, x_2) on line A between points a and b corresponds to some compromise between maximum transverse contact ratio and equal specific sliding. This choice is also interesting in practice.

If the gear design allows the center distance a_w to be increased from a geometrical point of view, the increase corresponds to the displacement of line A to line A', where it is tangent to the line ε_α = 1.2. This line (the curved line 1 in Fig. 3) limits the blocking contour from above and from the right. Note here that the minimum recommended value of the transverse contact ratio for spur gears according to Russian standards for their geometry calculation is ε_α = 1.2. Maximum values of x_Σ, a_w and α_{tw} are reached here within the blocking contour. Reference 5 shows that the contact of line A (a_w = constant) and ε_α = constant (line 1 in Fig. 3) always takes place at the intersection of lines 1 and B independent of the value ε_α. This is the point *c*, where the shift coefficients are x_1 = 0.497 and x_2 = 1.232, and the values of shifts sum coefficient, center distance and pressure angle are x_Σ = 1.729, a_w = 59.960 mm and α_{tw} = 26°42'33", respectively.



The abovementioned "Contour" CAD system also provides some additional possibilities for visualization of:

- tooth profiles of meshing pinion and gearwheel in any desired scale with many auxiliary elements, such as a line of action with marked out sections of single- and double-pair engagement, active sections of tooth profiles, arcs of relevant circumferences;
- gears in operation, i.e. rotating gearwheel and pinion, with all explaining geometric engagement attributes available;
- process of tooth cutting by means of a rack tool. Change of tool parameters and visualization of consequent changing geometry of a tooth being formed is available.

The mentioned additional capabilities, which are not always necessary for gear design in practice, may be useful for a designer and especially for educational purposes due to their possibilities of visualization.

Prospects for further development of the described approach are associated with implementation of some more auxiliary lines into the DBC structure. These lines include equal bending strength of a pinion and gearwheel, as well as a number of others. Prospects are also associated with application of the developed methodology of DBC construction for other types of gears: internal cylindrical, straight bevel, Novikov and others.

Conclusion

This paper gives a new approach to computer-aided design of spur and helical gears, based on implementation of dynamic blocking contours for choosing pinion and gearwheel shift coefficients in order to easily obtain some predetermined properties of a gear without preliminary calculations and complex optimizing procedures. The developed approach allows prediction of gear properties at the first stage of design, at the stage of initial data assignment.

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