# Relationship between Misalignment and Transmission Error in Cross-Axes Helical Gear Assemblies

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# QUESTION

I am a gear engineer for a motor manufacturer in China. I am writing about noise generated from cross-helical gear assembly error. I want to learn the relationship between the misalignment (center distance change and cross-angle shift) and transmission error. It is better under the loading and theory conditions. What is the trend of cross-helical gear development (use ZI worm and involute helical gear, point contact)?

**Expert Response Provided by** Dr. Hermann J. Stadtfeld. Crosshelical gears are manufactured by a generating process using a trapezoidal generating profile. In the question, the pinion is referred to as a "Z1 worm." A Z1 worm is paired with a worm gear. Although the worm has an involute profile, the worm gear pair has line contact and is not considered cross axes helical gearing. A Z1 worm gear pair is conjugate due to the fact that the worm gear is manufactured with a tool which resembles the mating worm. The following answer concentrates on cross axes helicals, where the contact and meshing conditions are quite different compared to those in worm gear drives. Figure 1

(left) shows a spur gear pair and (right) a 90° cross-axis helical gear pair.

Due to the cross-axes angle, the transverse involute profiles are in different planes and rotate about axes with different directions. In their theoretical position, the generating trapezoid can be placed between both members. As the generating element shifts, both members rotate without transmission error. This is a result of the kinematic coupling condition and not a result of the involute-based gearing law. Changes of the center distance eliminate the basis of the kinematic coupling condition, which then leads to transmission error. The larger the cross-axes angle, the larger is the transmission error caused by a center



Figure 1 Spur gear pair (left) and 90° cross angle helical gear pair (right).

distance change. The cross-axes angle helical pair is meshing in its theoretical position like a spur gear with extremely large length crowning.

The shaft angle error will cause edge contact and transmission error in spur gears. In the case of 90° shaft angled cross-axis helical gears, the effect of the shaft angle error is minimized because of the located point contact. Cross-axis helical gears have a virtual length crowning, which causes the point contact and reduces the sensitivity to shaft angle errors.

The following factors are unique to crossed-axes helical gears:

### Spur Gears

- Are special cases of cross-axes helical gears
- If the axes cross angle is zero, then the gears are spur gears
- The flank surfaces have line contact
- In this case the path of contact is oriented in profile direction (Fig. 2, bottom)
- The gearset is center distance-insensitive, but highly shaft angle-sensitive

### Introduction of Cross-Axes Angle

- If the axes cross angle is not equal to zero, then in the standard case both members have the same kind of helix, and a helix angle that is 50% of the shaft angle
- The axes cross angle is the sum of the two members' helix angle value

# Cross-Axes Angle 90°

- If the cross-axis angle is 90°, then the path of contact has the shape of a quarter circle (Fig. 2, bottom)
- The flank surfaces have point contact
- The gearset is sensitive to center distance changes, but has little sensitivity to shaft angle misalignment

# Small Cross-Axes Angle

- If the cross-axis angle is small (close to 0°), then the path of contact has the shape of a slim quarter ellipse (Fig. 2, center)
- The flank surfaces have point contact that will already spread under light load to contact lines
- The gearset is almost insensitive to center distance changes, but the sensitivity to shaft angle misalignment is medium to high

The bottom graphic in Figure 2 shows one contact line between generating rack and gear 1, and a second contact line between generating rack and gear 2. The contact lines are crossing under an angle and have only one common point. This is why crossed-axel helical gears have point contact rather than line contact. The sum of the intersecting points of gear 1 and gear 2 define the path of contact. Only the gear pair with zero-degree shaft angle in the top graphic has matching contact lines between gear 1, generating rack, and gear 2. Only contacting points along the theoretical path of contact (in red) transmit the correct ratio.

The relationship between generating rack and the two mating helical gears that mesh under a cross-axis angle is shown (Fig. 3). The two gear rotations, as well as the generating rack movement, are in three different planes. The graphic in Figure 3 makes it evident that the common contact location between pinion and gear in each shift position of the rack can only be a single point.



Figure 2 Crossed-axes helical gear relationships.



Figure 3 Generating rack between pinion and gear.

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Figure 4 shows in 4 steps the changes to the contact location and contact normal vectors in order to explain the physics behind the center distance sensitivity of cross-axes helical gears. The initial pair of normal vectors is drawn as an example in the pitch point. The center distance change moves the normal vector of the upper gear to location 2. The rotation 3 is required to eliminate the backlash that was caused by the center distance change. The new contacting point of both gears will neither be in position 1 or position 3. A new contacting point with a normal vector pair will result from the interaction of the two gear flank profiles (location 4). As a result, the initial path of contact (Fig. 4, left) is distorted to the green path of contact.

The different location of the path of contact in face width direction causes no direct transmission error. However, the different location in profile direction will create a transmission error. The reason is that the contact points on the two mating involutes that rotate in different planes lost their kinematic relationship. This relationship only exists if the two helical gears are located at their correct center distance, and the trapezoidal generating profile has contact with one helical gear at the top and with the mating helical gear at the bottom (Fig. 5).

The attempt to quantify the influence of a center distance change to the transmission error is shown (Fig. 5, left). A center distance change shows no influence if the cross-axes angle is zero. The transmission error increases with a parabolic characteristic as the shaft angle departs from zero. The transmission error reaches its maximum in case of 90° cross axes angle. The qualitative influence of an axes misalignment to the transmission error is shown (Fig. 5, right). The influence is the highest for the case of zero-degree shaft angle (spur gear) and diminishes as the shaft angle increases. The lowest point is reached at 90° shaft angle. Small, medium and large misalignments show equidistant graphs.

Although the diagrams are only qualitative, the relative relationship reflects the correct trends.

#### **Summary**

Crossed-axes helical gears have point contact and their surfaces are subject to high surface stress. The point contact can be explained with the orientation between generating rack, pinion and gear. A low transmission error can only be achieved in the low-load condition. Also the center distance insensitivity of spur and helical gears does not apply for crossed-axes helical gears (Fig. 5, left). However, the opposite characteristic can be seen regarding shaft misalignments. Spur and helical gears without length crowning react with high transmission error if the shafts are misaligned. Crossed-axes helical gears offer a "natural" length crowning, which reduces the sensitivity to small shaft angle changes (Fig. 5, right). 💽







Figure 5 Qualitative influence of center distance and shaft angle changes.

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