A Logical Procedure To Determine Initial Gear Size

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Abstract

A logical procedure is described for determining the minimum size gear set required to transmit a given load for a given operating life. Variations of this procedure are applied to different gear arrangements. These include speed reducers, speed increasers, star and planet reducers, and star and planet increasers. A logic flow diagram is included for computer application.

Introduction

When a gear set is to be designed for a new application, the minimum size gears with the required capacity are desired. These gears must be capable of meeting the power, speed, ratio, life, and reliability requirements.

The intent of this article is to present a logical and orderly sequence of steps to determine this size. The term "size" refers to the diameters and face widths. On multi-stage gear units this procedure would be applicable to one stage at a time.

For the sake of simplicity, it is assumed here that the gear set is steel and that the hardness of the meshing gears are alike and are given in the Rockwell C scale. Gears of differing hardness or gears on the BHN scale can also use this type of procedure. AGMA 218.01[1] provides equations to determine the allowable stresses.

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The late **ABRAHAM I. TUCKER** *was a consulting engineer specializing in gear-related engineering problems. He had experience in the design and development of high speed, high performance gear assemblies, hydrodynamic bearings for gearing and turbomachinery and lubrication systems for high speed machinery.*

A graduate of New York University, Mr. Tucker was the author of several technical papers. He was a Registered Professional Engineer and a member of ASME and Tau Beta Pi and Pi Tau Sigma honor societies.

Basic Equation

The basic equation for this procedure is derived from the classical Hertz equation for the surface compressive stress on two curved surfaces. It has been rearranged to produce a solution for a minimum pinion diameter.[2] For an external gear set it is

$$d \geqslant \left[\frac{0.7\ Tp\ E\ (mg + 1)\ (\cos HA)^2\ Kd}{Sc^2\ R\ mg\ \sin PAn\ \cos PAn\ mp}\right]^{1/3} \qquad (1)$$

For an internal gear set the term (mg −1) would be used in place of (mg +1).

Pinion Torque

The torque on the pinion determines its required size and, consequently, the size of the entire gear train. By definition, a pinion is the smallest member in a gear train. In a speed reducer it would be the driver. In a speed increaser it would be the driven member. In an epicycle train it may be either the sun or the planet (or star), depending on the ratio. Fig. 1 illustrates these differences. This will be considered in the equations which follow.

One of the major factors which determines the gear size is the transmitted torque through the pinion, Tp. Input data required to determine this torque are as follows:

1. Input speed, ni, rpm
2. Desired output speed, no, rpm
3. Horsepower, P
4. Number of equal power paths, K
5. Type of gear arrangement.

If the gear set is simple speed reducer, either single mesh or multiple countershaft, then:

$$Tp_1 = \frac{63025\ P}{ni\ K} \tag{2}$$

$$mg_1 = ni/no \tag{3}$$

If it is a simple speed increaser, then:

$$Tp_2 = \frac{63025\ P}{no\ K} \tag{4}$$

$$mg_2 = no/ni \tag{5}$$

For a speed reducer which has a star arrangement, where the input is to the sun and output is from the ring, and where $3 \leqslant Mg \leqslant 10$, the sun is the pinion. This will be designated as STAR 1. See Fig. 1. Then:

$$Mg = \frac{ni}{no} \tag{6}$$

$$mg_3 = \frac{Mg - 1}{2} \tag{7}$$

$$Tp_1 = \frac{63025\ P}{ni\ K} \tag{2}$$

When the star set reducer ratio is $1.2 \leqslant Mg < 3$, each star is a pinion. This will be designated as STAR 2. See Fig. 1. Then:

$$mg_4 = \frac{2}{Mg - 1} \tag{8}$$

$$Tp_3 = \frac{63025\ P}{ni\ mg\ K} \tag{9}$$

If the star set is a speed increaser and the input is to the ring, the output to the sun and the carrier would be fixed. Similar reasoning may be applied to determine which item is the pinion, its transmitted torque, and the diameter ratio of sun to star, mg.

For a planet type speed reducer where the ring is fixed, the input is to the sun and output is through the carrier. If the speed ratio is $4 \leqslant Mg \leqslant 11$, the sun is the pinion. This will be designated as PLANET 1. See Fig. 1. Then:

$$mg_5 = \frac{Mg - 2}{2} \tag{10}$$

$$Tp_1 = \frac{63025\ P}{ni\ K} \tag{2}$$

If the planet set speed ratio is $2.2 < Mg < 4$, the planets are the pinions. This will be designated as PLANET 2. See Fig. 1. Then:

$$mg_6 = \frac{2}{Mg - 2} \tag{11}$$

$$Tp_3 = \frac{63025\ P}{ni\ mg\ K} \tag{12}$$

If the planetary set is a speed increaser, the input is to the carrier, output is to the sun, and the ring gear is fixed.

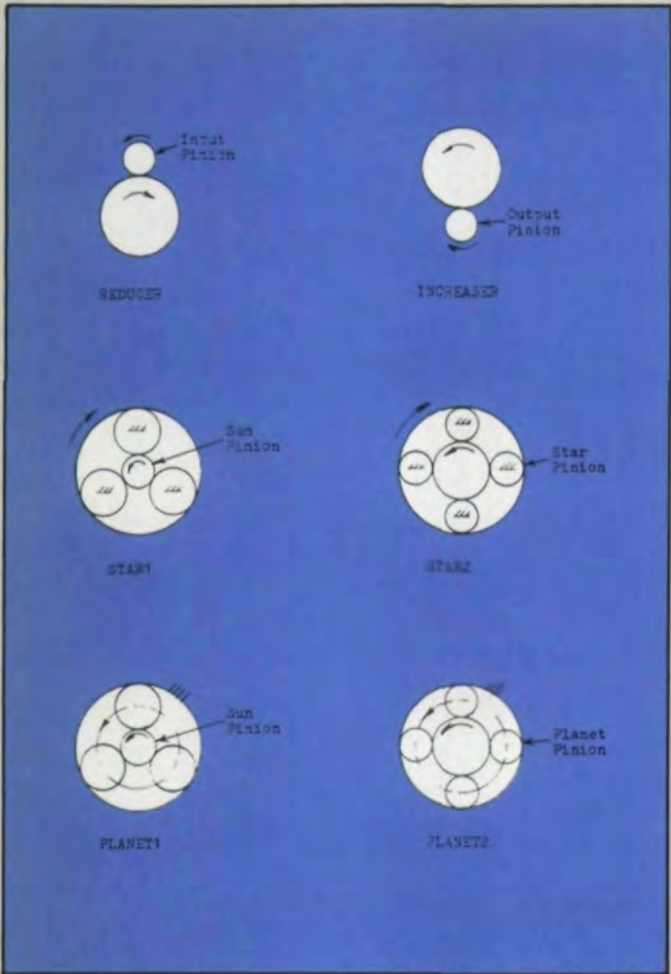

Fig. 1 – Some common gear arrangements showing the location of the pinion in the train.

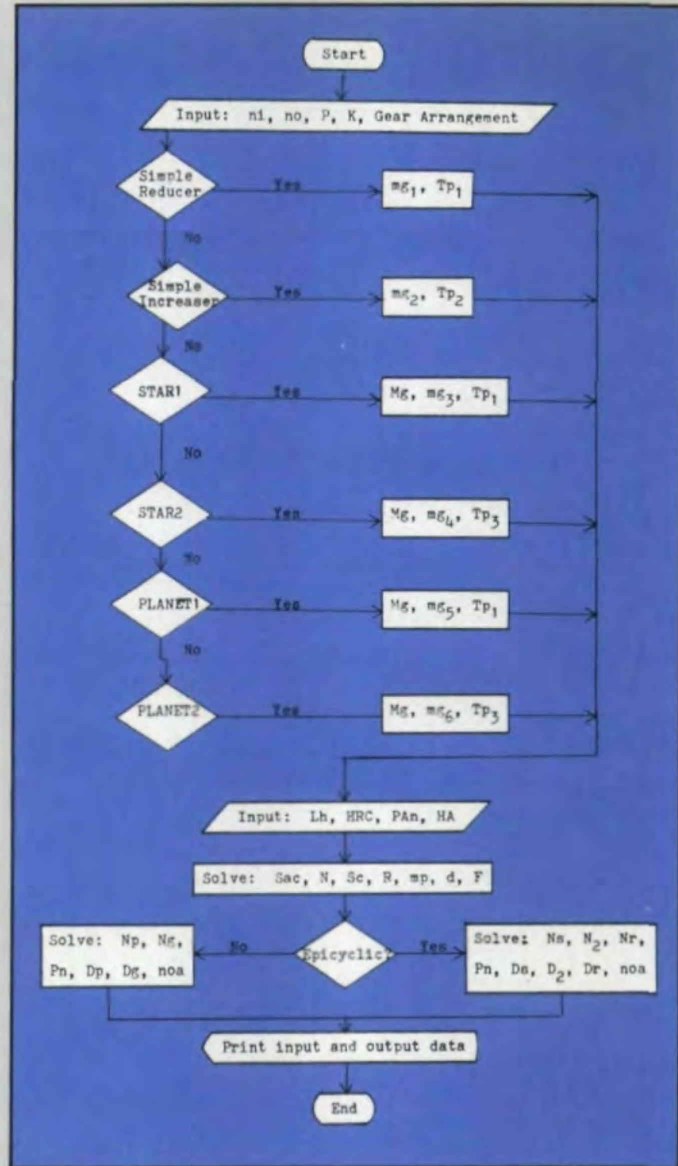

Fig. 2 – Flow Diagram.

Depending on the ratio, similar logic may be applied to determine which member is the pinion, its transmitted torque, and the diameter ratio of the sun and planet, mg.

Allowable Contact Stress

The allowable contact stress, Sc, for a given fatigue life in hours, Lh, is the next factor which determines the required gear size. To determine this allowable stress the input data needed are

1. Desired life in number of hours, Lh.
2. Material hardness, Rockwell C scale, HRC.

Allowable contact stresses at 10^7 cycles, Sac, are specified in AGMA 218.01, Table 5.[1] An equation to determine the conservative end of these allowable stresses would be

$$\text{Sac} = 3333.33\ \text{HRC} \qquad (13)$$

The life in hours must next be converted to number of contact cycles, N.

$$\text{N} = \text{Lh}\ 60\ \text{ni}\ \text{K for speed reducers} \qquad (14)$$

$$\text{N} = \text{Lh}\ 60\ \text{no for speed increasers} \qquad (15)$$

The maximum allowable stress from 0 to 10^4 cycles is constant. Therefore, if $N < 10^4$, then let $N = 10^4$.

Between 10^4 and 10^7 cycles, the allowable contact stress, Sc, to meet the fatigue life per AGMA 218.01 is

$$\text{Sc} = \text{Sac}\,\frac{2.466}{\text{Lc}^{.056}} \qquad (16)$$

Above 10^7 cycles, the allowable contact stress to meet the fatigue life is

$$\text{Sc} = \text{Sac}\,\frac{1.4488}{\text{Lc}^{.023}} \qquad (17)$$

Minimum Pinion Size

To determine the minimum pinion size, some additional input data are required.

A. For new designs, a reasonable ratio of F/d is desired. The face width should be short enough to minimize the effects of lead errors and shaft deflections. It should not be too short, or the required diameter would be too large.

Nomenclature

Dg = Gear pitch diameter
Dp = Pinion pitch diameter
Dr = Ring gear pitch diameter
Ds = Sun gear pitch diameter
D_2 = Star or planet pitch diameter
d = Minimum required pinion pitch diameter
E = Young's modulus
F = Face width
HA = Helix angle
HRC = Hardness, Rockwell C
IP = Integer part
K = Number of equal power paths
Kd = Derating factor
Lc = Life in number of cycles
Lh = Life in hours
Mg = Reduction ratio, epicyclic gear train
mg = Ratio of gear/pinion diameters
mp = Profile contact ratio
N = Number of life cycles
Ng = Number of teeth in gear
Np = Number of teeth in pinion
Nr = Number of teeth in ring gear
Ns = Number of teeth in sun
N_2 = Number of teeth in star or planet
Nri = Initial estimate of number of teeth in ring gear
ni = Input speed, rpm
no = Output speed, desired, rpm
noa = Output speed, actual, rpm
P = Horsepower
PAn = Pressure angle, normal
Pn = Diametral pitch, normal
PLANET 1 = A planetary set where $4 \leq Mg \leq 11$
PLANET 2 = A planetary set where $2.2 \leq Mg < 4$
R = Ratio, F/d
Sc = Allowable contact stress at the desired fatigue life, psi
Sac = Allowable contact stress at 10^7 cycles, psi
STAR 1 = A star gear set where $3 \leq Mg \leq 10$
STAR 2 = A star gear set where $1.2 \leq Mg < 3$
Tp = Pinion torque, lb. in.

A reasonable initial F/d ratio would be

$$R = \frac{mg}{mg + 1} \tag{18}$$

B. The normal pressure angle, PAn, and the helix angle, HA, must be provided as initial input data. They influence the surface curvature and the resultant surface compressive stress. A reasonable initial estimate for the profile contact ratio would be

$$mp = 2.54 - .04\ PAn \tag{19}$$

C. In the basic equation, mp is applicable only to helical gears. Therefore, for spur gears, $HA = 0$ and $mp = 1$.

D. An initial estimate for the total derating factor, Kd, is required. This estimate could be set to an initial value of 2. A more precise value could be determined after the size and pitch line velocity are known. The basic equation could then be reiterated.

This completes the initial input data. The minimum required gear size can now be determined using the basic equation:

$$d = \left[\frac{.7\ Tp\ 29.5\ 10^6\ (mg + 1)(\cos HA)^2\ Kd}{Sc^2\ R\ mg\ \sin PAn\ \cos PAn\ mp}\right]^{1/3} \tag{1}$$

and

$$F = d\ R \tag{20}$$

Numbers of Teeth and Diametral Pitch

When the minimum pinion diameter is determined, an integer number of teeth and a diametral pitch must be chosen. For simple meshes a practical number of pinion teeth, gear teeth, and diametral pitch may be determined by the following equations:

$$Np = \left[IP\ 17\ \frac{mg + 2}{mg} \cos HA + 1\right] \tag{21}$$

(Note: The above equation is an empirical determination which would normally provide teeth stronger in bending than in surface compression.)

$$Ng = IP\left[(Np\ mg) + .5\right] \tag{22}$$

$$Pn = IP\left[\frac{Np}{d \cos HA}\right] \tag{23}$$

Final pitch and gear pitch diameters would be

$$Dp = \frac{Np}{Pn \cos HA} \tag{24}$$

$$Dg = \frac{Ng}{Pn \cos HA} \tag{25}$$

For star or planet trains, an empirical equation to determine a reasonable number of pinion teeth would be

$$Np = IP\left[17 \frac{mg + 1}{mg} \cos HA + 1\right] \tag{26}$$

Then:

$$Pn = IP\left[\frac{Np}{d \cos HA}\right] \tag{23}$$

To determine numbers of teeth in the other gears in star or planet sets, the following procedures are required:

1. For a "STAR 1" or "PLANET 1" reducer the sun gear is the pinion. Therefore:

$$Ns = Np \quad (27)$$

2. For a "STAR 2" or "PLANET 2" reducer the stars or planets are the pinions. Therefore:

$$Ns = IP\left[(Np\ mg) + 1\right] \quad (28)$$

3. For all star reducers, an initial estimate for number of ring gear teeth would be

$$Nri = Ns\ Mg \quad (29)$$

4. For all planet reducers an initial estimate for number of ring gear teeth would be

$$Nri = Ns(Mg - 1) \quad (30)$$

The sum of the teeth in the sun and ring must be divisible by the number of stars or planets. The result must be an integer. The following procedure will accomplish this.

$$X = IP\left[\frac{Nri + Ns}{K} + .5\right] \quad (31)$$

$$Nr = K\ X - Ns \quad (32)$$

$$N_2 = \left[IP\ \frac{Nr - Ns}{2}\right] \quad (33)$$

Pitch diameters of the gears in the epicyclic train would then be

$$Ds = \frac{NS}{Pn \cos HA} \quad (34)$$

$$D_2 = \frac{N_2}{Pn \cos HA} \quad (35)$$

$$Dr = \frac{Nr}{Pn \cos HA} \quad (36)$$

With the numbers of teeth now known, actual output speed, noa, and the precise reduction ratio, Mg, may be determined.

The procedures above for determining numbers of teeth and diametral pitch will provide gear sets with pinions slightly larger than the required minimum. This approach is intentionally conservative.

A logic flow diagram of this gear size determination is shown in Fig. 2. It may be used as a guide to program this procedure on a computer.

Conclusion

A complete and logical procedure can be applied to determine the size of a gear set to carry a given load for a given required life. The examples herein show the procedures for applying this logic to several varieties of gear trains. It can be expanded to include many additional types of gear train arrangements, such as epicyclic speed increasers. The steps are in a logical sequence to provide easy programming for a computer or on a calculator. They will prove useful to the design engineer who is given a complete set of requirements or specifications and must then design a suitable gear set starting with a blank piece of paper.

References

1. AGMA 218.01 – AGMA Standard for Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth. American Gear Manufacturers Association, Arlington, Virginia.
2. TUCKER, A. I., ASME Paper 80-C2/DET-13, The Gear Design Process.

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GEAR TECHNOLOGY wishes to thank Mrs. Mona Tucker for permitting us to use this article.

TECHNICAL CALENDAR

November 11-13, 1986
SME Gear Processing and Manufacturing Clinic
Chicago, IL

Call for Papers: The Society of Manufacturing Engineers has issued a call for papers for this meeting. Please submit all papers on or before July 15th. The clinic will also include vendor tabletop exhibits.

For more information, contact Joseph A. Franchini at SME (313) 271-1500, ext. 394.

November 19-21, 1986
Seminar: Gear System Design for Minimum Noise
University of Wisconsin

This course provides an overview of noise control in gear system design and offers practical design information with a minimum of acoustical theory. The course is for engineers who design or specify gears or gear drives.

For additional information contact:
John Leaman at (414) 224-4189.

1987 GEAR MANUFACTURING SYMPOSIUM— CALL FOR PAPERS

The American Gear Manufacturers' Association is issuing a call for papers for its 1987 Gear Manufacturing Symposium to be held April 12-14, 1987, at the Hyatt Regency Hotel, Dearborn, MI. Papers on basic gearing, traditional processes, quality control, materials and emerging technologies will be accepted. Authors are asked to submit abstracts as soon as possible.

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